APPENDIX 3 - MOVEMENT OF CONTAMINANTS IN AQUIFERS

When an oil or gas reservoir is situated below an aquifer, drillers have to penetrate through the water bearing strata to get to the petroleum deposit. Usually all the necessary steps are taken to make sure that drilling muds and chemicals in the mud do not flow from the well to the aquifer. But, in case of a very porous lost circulation zone, a possibility exists that mud could seep through to the aquifer. In this case, environmental engineers must answer two important questions:

1) If the drilling mud enters the aquifer, to what extent will the aquifer be contaminated during the time the drill bit is penetrating the aquifer? 2) After the well penetrates through the aquifer, and after it is cased and cemented, no contaminant seeps through from the well, but the volume of pollutant already in the aquifer is transported by water under the effect of pressure differential created by pumping or gravity. The problem to resolve then would be to find what happens to the contaminant that has already invaded the aquifer.

As stated above, the history of contaminant movement in the aquifer from time zero until the final distribution goes through two phases: Phase I deals with the transient movement of the contaminant during the drilling phase and Phase II tackles the problem of transient contaminant distribution after the well is cemented along its interface with the aquifer.

Assumptions

Both Phases I and II are subject to the following assumptions:

a. No chemical or physical retardation of the contaminant
b. No transverse dispersion
c. Isotropic homogeneous medium (aquifer)
d. Contaminants carried by convection and dispersion
e. Dispersion is a diffusive process
f. The well penetrates the entire thickness of the aquifer within two days.
Phase I

As indicated above, a worst case scenario is obtained by maintaining at the interface between the well and the aquifer a concentration of contaminant equal to the maximum percentage of additives present in the drilling mud. If this concentration is designated by $S$, and if the interface between the well and the aquifer is taken as the spatial origin of the problem, one of the boundaries of the problem would be:

$$\text{For } x = 0, \quad S(0,t) = 0 \quad \text{for all } t$$  \hspace{1cm} (1)

Far away from this point or at a distance equal to the radius of drainage of the considered well it is safe to assume that the contaminant concentration is nil. This can be expressed as

$$\text{For } x = 76200 \text{ cm}, \quad S(76200,t) = 0$$  \hspace{1cm} (2)

$$\text{for all } t$$

Before any drilling takes place or before the well penetrates the aquifer, there is no contaminant in the aquifer or,

$$\text{For } t = 0, \quad S(x,0) = 0 \quad \text{for all } x$$  \hspace{1cm} (3)

Between boundaries (1), (2) and (3) the value of contaminant concentration is given by the function $S(x,t)$ which is the solution of the linear partial differential equation

$$\frac{\partial S}{\partial t} (x,t) + U \frac{\partial S}{\partial x} (x,t) = D \frac{\partial^2 S}{\partial x^2} (x,t)$$  \hspace{1cm} (4)

This equation represents the transient transport of a substance $S$ subject to a constant velocity of flow $U$ and constant longitudinal dispersion $D$. The medium of flow is assumed homogeneous. It is assumed that transversal dispersion is negligible as verified by analyzing several sets of data gathered from the literature. These data show that under the conditions considered the transversal dispersion represents only 5 percent of the longitudinal component.

The solution of equation (4), subject to boundaries (1), (2) and (3) was given by Rifai, Kaufman and Todd \cite{rife} and Ogata and Banks \cite{ogat}.

$$\frac{S(x,t)}{S_0} = \frac{1}{2} \ erfc \left( \frac{x - Ut}{2(\Delta t)^{1/2}} \right) + \frac{1}{2} \ e^{\frac{UX}{D}} \ erfc \left( \frac{x + Ut}{2(\Delta t)^{1/2}} \right)$$  \hspace{1cm} (5)
Relation (5) allows the calculation of the concentration of contaminant as a function of distance x and time t. This solution was applied to the Jamestown aquifer. More specifically, it was used to study the percolation and movement of the additives contained in the mud into the aquifer during a period of two days while the oil and gas well was penetrating the water saturated interval.

Mud Composition - Maximum Concentration

The maximum concentration of additives in the mud was obtained by summing up all the material added to a volume of 250 barrels of water and then dividing the sum by the total sum of additives and water. Our regional office in Olean reported the following mud composition:

- 100 sacks of bentonite each weighing 50 lbs.
- 1 sack of lime each weighing 80 lbs.
- 1/2 sack of caustic soda weighing 25 lbs.
- 10 sacks of cottonseed hull each weighing 40 lbs.
- 5 sacks of mud seal each weighing 60 lbs.
- 250 barrels of water

The original concentration of contaminant $S_0$ may be obtained as follows:

$$S_0 = \frac{100 \times 50 + 80 \times 1 + 0.5 \times 25 + 10 \times 40 + 5 \times 60}{(5000 + 80 + 12.5 + 400 + 300) + 250 \times 62.4} = 6.21\%$$

Results Obtained

Equation (5) gives the concentration distribution of the contaminant as a function of time and distance. One useful way to use relation (5) would be to fix a certain time t, hold it constant and evaluate S for various positions of x until S becomes equal to zero. These calculations give the concentration profile for the assumed time. The integration of this profile gives the volume or mass of contaminant which has entered the aquifer up to the assumed time t. If it took 2 days to penetrate the aquifer, then the concentration profile for $t = 2$ days would be calculated as a function of x.

The result of this work is shown in Fig. 1. which indicates that about 280 cms. of the aquifer have been contaminated with a concentration of 6.2 percent. This distance represents only 0.39% of the drainage radius. The remaining contaminated part represents a distance of 120 cms. with concentration decreasing from 6.2 percent to zero.
APPENDIX 3, FIGURE 1  CONTAMINANT PENETRATION INTO AQUIFER

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CONCENTRATION (%) vs. DISTANCE (CM)

2 DAYS
**Phase II**

After 2 days of drilling it was assumed that the gas well has already penetrated the entire thickness of the aquifer and that a good cementing job has eliminated completely the infiltration of the contaminant from the wellbore. Stated in mathematical terms this condition can be translated into

\[ \text{For } X = 0, \quad S(0, t) = 0 \quad \text{For all } t \]  

(6)

The second boundary condition at the end of the drainage radius is similar to the condition used in Phase 1 or

\[ S(74200, t) = 0 \quad \text{For all } t \]  

(7)

In this phase the most important condition is the initial boundary. At \( t=0 \) the profile for \( t=2 \) days would be used

\[ \text{For } 0 < X < 280 \quad S(X, 0) = 6.2 \]  

(8)

For \( 280 < X < 74200 \) The remaining part of curve \( t=2 \) days was curve filled—See Figure 1.

In brief, the problem to solve in this second phase would consist of solving equation (4) subject to boundary and initial conditions (6), (7) and (8).

Because of the complexity of the initial condition, equation (5) was solved by means of an explicit numerical scheme. This scheme is based on the following approximations:

\[ \frac{2s}{2t} \approx \left( S_{i+1, n+1} - S_{i, n} \right) / \Delta t \]  

(9)

\[ \frac{2s}{2x} \approx \left( S_{i+1, n} - S_{i-1, n} \right) / 2 \Delta x \]

\[ \frac{2^2s}{2x^2} \approx \left( S_{i+1, n} - 2S_{i, n} + 5S_{i-1, n} \right) / \Delta x^2 \]

Replacing the approximations

\[ \frac{2s}{2t}, \quad \frac{2s}{2x} \quad \text{And} \quad \frac{2^2s}{2x^2} \]

by their values in equation (4) and making the necessary algebraic transformations and ordering, one can readily obtain the following numerical relations:

\[ S_{i, n+1} = S_{i-1, n} \left[ D \Delta t + \frac{U}{A} \Delta x \Delta t \right] \frac{1}{\Delta x^{-2}} \]

(10)

\[ + \quad S_{i, n} \left[ (\Delta x^2 - 20 \Delta x) \right] \frac{1}{\Delta x} \]

\[ + \quad S_{i+1, n} \left[ D \Delta t - \frac{U}{A} \Delta x \Delta t \right] \frac{1}{\Delta x^2} \]
Relation (10) states that the value of the concentration at position \( i \) and time \( t + t \) can be expressed explicitly in terms of 3 values of concentration calculated at time \( t \) but at positions \( i-1, i \) and \( i + 1 \). Because such a scheme can be unstable the following conditions must be imposed on the coefficients of relations (10).

\[
\frac{\Delta t}{\Delta x^2} \leq \frac{i}{20}
\]

\[
\Delta x \leq \frac{20}{v}
\]

A Fortran computer program was written and debugged to solve equation (10) subject to the boundary conditions expressed in (6), (7) and (8). The increments of space and time were carefully chosen subject to the constraints of (11) to ascertain that a stable solution was obtained.

Several runs were made with variable coefficients of dispersion with different results as discussed in the next section.

Results Obtained

The manner in which contaminants move through an aquifer depends largely on the value of the coefficient of dispersion and the velocity of the water flow in the aquifer. A review of the literature revealed that under ideal laboratory conditions and homogeneous media, the coefficient of dispersion at low flow velocities is of the same order of magnitude as the velocity itself. However, as the velocity of water flow increases, the coefficient of dispersion increases much more rapidly.

In flow through heterogeneous formations, such as occurs in practical cases, tortuosity and a moderate to high velocity of water flow result in a large increase in the coefficient of dispersion. For these conditions, the effects of dispersion control the distribution of contaminants much more effectively than convection.

Two sets of graphs were prepared: one with a velocity of flow and a coefficient of dispersion respectively equal to 0.019 cm/sec and 0.019 cm²/sec; the other with an equal velocity but a coefficient of dispersion ten times larger (see Figures 2 & 3). As expected, the run with the higher dispersion yielded a set of curves with lower peaks but larger breadths.
APPENDIX 3, FIGURE 2  DISTRIBUTION OF CONTAMINANT IN AQUIFER

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$U = 0.0019 \text{ cm/sec}$

$D = 0.019 \text{ cm}^2 / \text{sec}$

CONCENTRATION (%)

DISTANCE (CM)

THOUSANDS

10 DAYS  20 DAYS  30 DAYS  40 DAYS
APPENDIX 3, FIGURE 3

DISTRIBUTION OF CONTAMINANT IN AQUIFER

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CONCENTRATION (%)

DISTANCE (cm)

10 DAYS  20 DAYS  30 DAYS  40 DAYS

U = 0.0019 cm/sec
D = 0.19 cm²/sec
For example, for $t=10$ days and $D=0.019 \text{ cm}^2/\text{sec}$, the concentration of contaminant is 4.5 percent. The zone of finite concentration extends about 1500 cms. For $D=0.019 \text{ cm}^2/\text{sec}$, the peak concentration of contaminant decreases to about 1.2 percent but the size of the zone of finite concentration increases to about 4000 cms. If $t$ is increased to 20, 30, and 40 days, the peak concentration for $D=0.019 \text{ cm}^2/\text{sec}$ decreases from 4.5 percent to 3.5, 3 and 2.5 percent. For equal values of time, the peak concentration corresponding to $D=0.19 \text{ cm}^2/\text{sec}$ decreases from 1.2 to 0.8 to 0.7 to 0.6 percent.

From a practical standpoint, the way to circumvent this type of pollution would be to conduct a careful evaluation of the dispersion in the aquifer under consideration. If the dispersion is small, the zone of contamination moves as a slug with the same velocity as the water in the aquifer. This slug can be produced out of the aquifer over a short interval of time. However, if the coefficient of dispersion is larger, practical solution dictates that the water well be located far enough from the drilling well so that the contaminant concentration falls below the concentration allowed by Federal and State regulations.

From this analysis, one can conclude that for the type of dispersion likely to exist in porous media and for a large enough water velocity in aquifer, a worst case scenario indicates that the contaminant concentration is too small to cause any harmful effects.

The case considered in this study was labeled a worst case scenario. In reality it should be labeled an unrealistic worst case. The reason is simply that as soon as mud enters the aquifer, bentonite and other contaminants presents in the mud are usually deposited near the wellbore plugging the formation and isolating the aquifer from further mud flow.
REFERENCES
